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**A WEIBULL CHARACTERIZATION FOR TENSILE  
FRACTURE OF MULTICOMPONENT BRITTLE FIBERS**

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TECHNICAL PAPER to be presented at the  
Fall Metals Conference  
sponsored by the Metallurgical Society of the American Institute  
of Mining, Metallurgical, and Petroleum Engineers  
Chicago, Illinois, October 24-27, 1977

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Abstract

E-9357

Necessary to the development and understanding of brittle fiber reinforced composites is a means to statistically describe fiber strength and strain-to-failure behavior. A statistical characterization for multicomponent brittle fibers is presented. The method, which is an extension of usual Weibull distribution procedures, statistically considers the components making up a fiber (e.g., substrate, sheath, and surface) as separate entities and taken together as in a fiber. Tensile data for silicon carbide fiber and for an experimental carbon-boron alloy fiber are evaluated in terms of the proposed multicomponent Weibull characterization.

Introduction

Fiber reinforced composites are being considered for future high performance materials applications. Excellent strength/density and stiffness/density properties are associated with composite materials. Reinforcing fibers include both ductile and non-ductile (brittle) materials. Tungsten and steel are examples of the former and boron, silicon carbide, carbon, single crystal and polycrystalline alumina, and glasses the latter. Only brittle reinforcing fibers, which are typically low in density and have high strength and modulus over a wide range of temperatures are considered herein. Incorporating these lightweight fibers into a suitable matrix offers the interesting proposition of utilizing low density materials which have superior strength and stiffness compared to conventional monolithic metals.

Brittle fiber ultimate tensile strength can be described statistically and composite strength theories incorporate statistical fiber strength characterizations (1-5). The Weibull distribution function (6,7), which incorporates a size-strength effect, is often

used to characterize the strength of brittle fibers. The usual Weibull characterization, however, has been found deficient for some brittle fiber materials (3, 8-10), and has provided little insight for fiber development and improvement purposes.

It is the primary purpose here to present and demonstrate an elaboration of the usual Weibull characterization for multicomponent brittle fibers. The method considers the components making up a fiber (e.g., substrate, chemical vapor deposited sheath, surface, etc.) separately and together (assembled into a composite fiber). The characterization permits an improved description of brittle fiber strength and is of instructional value in the interpretation of brittle fiber tensile strength data. It is believed the characterization will aid in the identification of those portions of a fiber to which, if improvements were made, significantly improved total fiber performance would (or would not) result.

Room temperature tensile data are presented for two candidate reinforcing fibers: silicon carbide fiber of 142  $\mu\text{m}$  (0.0056 in.) diameter and a carbon-boron alloy fiber developed and evaluated under NASA contracts (11-15). Silicon carbide tensile data of Reference 8 are also interpreted in terms of the multicomponent statistical characterization.

### Theoretical Considerations

The Weibull function is often used to characterize strength properties of brittle materials, including brittle reinforcing fibers. As shown below the Weibull characterization predicts a size-strength relationship, that is, mean ultimate tensile strength (UTS) and/or mean strain-to-failure (STF) decrease as the volume or surface area increase. Predictions of the Weibull function are summarized and are followed by the development of the statistical characterization for multicomponent fibers.

### Weibull Single-Component (WSC) Characterization

The following relationships are as outlined by Corten (16) but in terms of strain rather than stress. The cumulative Weibull distribution function for STF,  $\epsilon$ , of a single component brittle fiber is:

$$G(\epsilon) = 1 - \exp \left[ -\omega \left( \frac{\epsilon - \epsilon_u}{\epsilon_o} \right)^m \right] \quad \text{for } \epsilon \geq \epsilon_u$$
$$= 0 \quad \text{elsewhere} \quad (1)$$

where

$G(\epsilon)$  the probability that the fiber will fail at a strain  $\leq \epsilon$

$\epsilon_u$  Weibull location parameter representing the strain at or below which the probability of failure is zero. It is noted here that this term will be identified with residual compressive strain in the multicomponent statistical approach described below

$\epsilon_o$  Weibull scale parameter

$m$  Weibull shape parameter

$\omega$  dimensionless value representing the number of incremental volumes or surfaces in a test gage length

The  $\omega$  term is somewhat analogous to the number of links in a chain. In this paper  $\omega$  will be used to designate either the number of incremental volumes or incremental surfaces in a test gage length. Since the value of the parameter  $\epsilon_o$  is dependent on the choice of incremental volume or surface, the increment sizes must be defined and specified and used throughout a given investigation. In this paper an incremental volume  $\alpha$  shall be  $1.6387 \times 10^{-5} \text{ cm}^3$  ( $10^{-6} \text{ in}^3$ ) and an incremental surface  $\gamma$  shall be  $6.4516 \times 10^{-3} \text{ cm}^2$  ( $10^{-3} \text{ in}^2$ ). For volume and surface respectively, the  $\omega$  values are:

$$\omega = \frac{\text{volume}}{\alpha}$$

$$\omega = \frac{\text{surface area}}{\gamma}$$

For example, a 0.01 cm diameter fiber of 2.54 cm gage length will have an  $\omega$  value for volume of 12.17. Similarly, the same fiber will have an  $\omega$  value for surface of 12.37.

The probability density function for the above is:

$$g(\epsilon) = \frac{\omega m}{(\epsilon - \epsilon_u)} \left( \frac{\epsilon - \epsilon_u}{\epsilon_o} \right)^m \exp \left[ -\omega \left( \frac{\epsilon - \epsilon_u}{\epsilon_o} \right)^m \right] \quad \text{for } \epsilon \geq \epsilon_u$$

$$= 0 \quad \text{elsewhere} \quad (2)$$

The expected mean STF of fibers tested one at a time is given by:

$$\bar{\epsilon} = \int_{\epsilon_u}^{\infty} \epsilon g(\epsilon) d\epsilon$$

$$= \epsilon_o \omega^{-1/m} \Gamma \left( \frac{m+1}{m} \right) + \epsilon_u \quad (3)$$

and the variance for fiber STF is:

$$\begin{aligned}
S_{\epsilon}^2 &= \int_{\epsilon_u}^{\infty} (\epsilon - \bar{\epsilon})^2 g(\epsilon) d\epsilon \\
&= \epsilon_0^2 \omega^{-2/m} \left[ \Gamma\left(\frac{m+2}{m}\right) - \bar{\epsilon}^2 \Gamma\left(\frac{m+1}{m}\right) \right]
\end{aligned} \tag{4}$$

where  $S_{\epsilon}$  is the standard deviation and  $\Gamma[(m+1)/m]$  and  $\Gamma[(m+2)/m]$  are tabulated gamma functions.

The coefficient of variation (CV) is:

$$CV = \frac{S_{\epsilon}}{\bar{\epsilon}} \tag{5}$$

In the statistical characterization of brittle fibers the location parameter  $\epsilon_u$  is often set equal to zero and as such the CV will be a function only of the  $m$  parameter and is independent of  $\omega$ . For all positive values of  $\epsilon_u$ , the CV decreases with increasing  $\omega$ .

According to equations (3) and (4), both  $\bar{\epsilon}$  and  $S_{\epsilon}$  decrease with increasing test section volume or surface, represented by the term  $\omega$ . Thus at a given fiber diameter,  $\bar{\epsilon}$  and  $S_{\epsilon}$  are predicted to decrease for increasing gage lengths. Equations (3) and (4) also predict that for a given gage length,  $\bar{\epsilon}$  and  $S_{\epsilon}$  should decrease with increasing fiber diameter. For tensile data that can be described by the WSC characterization, a plot of  $\ln(\bar{\epsilon} - \epsilon_u)$  versus  $\ln(\omega)$  gives a straight line of slope equal  $-1/m$ . If tensile data are for a single fiber diameter, a plot of  $\ln(\bar{\epsilon} - \epsilon_u)$  versus  $\ln(\text{gage length})$  also gives a straight line of slope equal  $-1/m$ .

#### Weibull Multicomponent (WMC) Characterization for Brittle Fibers

In the WMC characterization a fiber is considered to consist of one or more components, each having unique physical properties and flaw distributions. Components are either volume elements (e.g., a substrate or a chemical vapor deposited (CVD) sheath) or surface elements (e.g., a fiber surface or an interface between the substrate and a CVD sheath). It is assumed that the STF behavior of each component can be described by an appropriate statistical characterization.

Wawner (17) reported fracture origins for boron-on-tungsten fiber and characterized their location as to substrate, substrate/CVD sheath interface, bulk CVD sheath, and fiber surface. Smith (18), who also tested boron-on-tungsten fiber, suggested that most tensile fractures originated in the fiber core (tungsten boride). Crane and Krukons (19) reported that for silicon carbide-on-tungsten fiber, tensile fractures initiated primarily at the tungsten/SiC sheath interface and at the fiber surface. McHenry and Tressler (20) indicate that most SiC-on-carbon fiber tensile

failures originate at the carbon-SiC interface. In the context of the WMC characterization, surface (or interface) and volume components have been identified as sites of fracture initiation in real fibers. This suggests that by considering the different fracture sites explicitly, the WMC characterization could lead to better failure predictions than the usual single-component Weibull description.

The WSC characterization is used here to describe STF behavior of each component. For a brittle fiber made up of  $n$  components it is proposed that the fiber will fail in longitudinal strain due to activation of a flaw in any or all of the  $n$  components. Assuming independence of flaw activation in each component, the cumulative distribution function,  $Q(\epsilon)$ , for fiber fracture will be:

$$Q(\epsilon) = 1 - \prod_{i=1}^n [1 - G_i(\epsilon)] \quad (6)$$

where  $G_i(\epsilon)$  is equation (1) for the  $i^{\text{th}}$  component. Substitution from equation (1) into equation (6) gives for an  $n$  component fiber:

$$Q(\epsilon) = 1 - \exp \left[ - \sum_{i=1}^n \omega_i \left( \frac{\epsilon - \epsilon_{u_i}}{\epsilon_{o_i}} \right)^{m_i} \right] \quad \text{for } \epsilon \geq \text{lowest valued } \epsilon_{u_i}$$

$$= 0 \quad \text{elsewhere} \quad (7)$$

Note:

Individual  $\omega_i \left( \frac{\epsilon - \epsilon_{u_i}}{\epsilon_{o_i}} \right)^{m_i}$  terms are zero in the above and following multicomponent relationships for  $\epsilon$  less than or equal  $\epsilon_{u_i}$ .

The probability density function for the  $n$  component case is:

$$q(\epsilon) = \left[ \sum_{i=1}^n \frac{m_i \omega_i}{(\epsilon - \epsilon_{u_i})} \left( \frac{\epsilon - \epsilon_{u_i}}{\epsilon_{o_i}} \right)^{m_i} \right] \exp \left[ - \sum_{i=1}^n \omega_i \left( \frac{\epsilon - \epsilon_{u_i}}{\epsilon_{o_i}} \right)^{m_i} \right]$$

$$\quad \text{for } \epsilon \geq \text{lowest valued } \epsilon_{u_i}$$

$$= 0 \quad \text{elsewhere} \quad (8)$$



The mean failure strain, the variance, and the coefficient of variation for the composite fiber are given by:

$$\bar{\epsilon} = \int_Y^{\omega} \epsilon q(\epsilon) d\epsilon \quad (9)$$

$$S_{\epsilon}^2 = \int_Y^{\omega} (\epsilon - \bar{\epsilon})^2 q(\epsilon) d\epsilon \quad (10)$$

$$CV = \frac{S_{\epsilon}}{\bar{\epsilon}} \quad (11)$$

The lower limit for integration  $Y$  is the lowest valued  $\epsilon_{u_i}$ . Equations (9) and (10) cannot be solved explicitly, but can be evaluated by computer numerical integration methods.

Since fiber UTS is a property more often measured and reported than fiber STF, it is desirable to have expressions such as equations (7) to (11) in terms of fiber UTS. A change in variable from fiber strain to fiber stress,  $\sigma$ , is accomplished using the Rule-of-Mixtures (ROM) relationship for composite fiber modulus  $E_f$ :

$$\epsilon = \frac{\sigma}{E_f} = \frac{\sigma}{\sum_{i=1}^n V_i E_i} \quad (12)$$

$V_i$  and  $E_i$  are the volume fraction and modulus, respectively, for the  $i^{\text{th}}$  component. Cumulative distribution and probability density functions in terms of fiber stress become:

$$Q(\sigma) = 1 - \exp \left[ - \sum_{i=1}^n w_i \left( \frac{\sigma - E_f \epsilon_{u_i}}{E_f \epsilon_{o_i}} \right)^{m_i} \right]$$

for  $\sigma \geq \text{lowest valued } E_f \epsilon_{u_i}$

elsewhere (13)

= 0

$$q(\sigma) = \left[ \sum_{i=1}^n \frac{m_i \omega_i}{(\sigma - E_f \epsilon_{u_i}) \left( \frac{\sigma - E_f \epsilon_{u_i}}{E_f \epsilon_{o_i}} \right)^{m_i}} \right] \exp \left[ - \sum_{i=1}^n \omega_i \left( \frac{\sigma - E_f \epsilon_{u_i}}{E_f \epsilon_{o_i}} \right)^{m_i} \right]$$

for  $\sigma \geq$  lowest valued  $E_f \epsilon_{u_i}$

$$= 0 \quad \text{elsewhere} \quad (14)$$

Relationships for the mean and standard deviation in terms of stress are as in equations (9) and (10) but with incorporation of the change of variable. Alternately, it is simpler to multiply solutions of equations (9) and (10) by the fiber modulus. The CV will be identical in value for either stress or strain.

The WMC characterization contains a feature useful in correlating experimental observations of fiber fracture origins. Given that a large fiber sample has been tensile tested to failure at a given gage length and fiber diameter, the fraction of the fiber failures whose origin was in the  $j^{\text{th}}$  component can be estimated. From equation (6), the probability that the  $n$ -component fiber will fail between  $\epsilon$  and  $\epsilon + d\epsilon$  is:

$$\frac{dQ(\epsilon)}{d\epsilon} d\epsilon = q(\epsilon) d\epsilon = \sum_{j=1}^n g_j(\epsilon) \prod_{i \neq j} [1 - G_i(\epsilon)] d\epsilon \quad (15)$$

where  $i$  and  $j$  subscripts refer to components. Each summation term is interpreted as the probability that a flaw in component  $j$  initiated fiber failure between  $\epsilon$  and  $\epsilon + d\epsilon$ . For the  $j^{\text{th}}$  term, substitution from equations (1) and (2) gives:

$$g_j(\epsilon) \prod_{i \neq j} [1 - G_i(\epsilon)] d\epsilon = \frac{m_j \omega_j}{(\epsilon - \epsilon_{u_j}) \left( \frac{\epsilon - \epsilon_{u_j}}{\epsilon_{o_j}} \right)^{m_j}} \cdot \exp \left[ - \sum_{i=1}^n \omega_i \left( \frac{\epsilon - \epsilon_{u_i}}{\epsilon_{o_i}} \right)^{m_i} \right] d\epsilon \quad (16)$$

Given that a large sample has been tensile tested to failure, the following is the expected fraction of all fractures (over the entire strain range) that had origin in component  $j$  and which occurred in the strain range  $\epsilon_1$  to  $\epsilon_2$  ( $\epsilon_1 < \epsilon_2$ ).

$$Z_j = \int_{\epsilon_1}^{\epsilon_2} \frac{m_j \omega_j}{(\epsilon - \epsilon_{u_j})} \left( \frac{\epsilon - \epsilon_{u_j}}{\epsilon_{o_j}} \right)^{m_j} \exp \left[ - \sum_{i=1}^n \omega_i \left( \frac{\epsilon - \epsilon_{u_i}}{\epsilon_{o_i}} \right)^{m_i} \right] d\epsilon \quad (17)$$

For a single component brittle material which is free of imposed or residual strain and whose strength must be characterized statistically, it can be assumed that the parameter  $\epsilon_u$  in the above relationships should be zero. The term,  $\epsilon_u$ , however, will be identified with and defined as residual compressive strain. Residual stresses and strains often occur in portions of a composite fiber which result from, among other factors, thermal expansion differences and/or volume changes upon reaction between the substrate and the CVD sheath. An example is boron fiber which is produced by CVD of boron onto a tungsten substrate. The resultant substrate core is in a residual compressive stress state following fabrication (17,21). In such a case, fiber failure would not occur due to substrate flaws until the fiber were strained to such an extent that the substrate became unloaded. This behavior can be accommodated in the WMC characterization by defining the  $\epsilon_{u_i}$  terms as residual compressive strain in the respective  $i^{\text{th}}$  component (e.g., substrate). Therefore, the  $\epsilon_u$  parameter, for purposes here, is not a statistical curve fitting parameter but rather, a mechanical property peculiar to a given component of the particular fiber being considered. Note that to retain conventional sign notation in the Weibull relationships, a residual compressive strain has a positive sign when identified with the  $\epsilon_u$  parameter.

To summarize, the WMC characterization for brittle fiber tensile behavior is a function of component volume and/or surface area, component moduli, component residual strain, and the STF behavior given by the Weibull parameters  $m$  and  $\epsilon_o$  for each component. Predictions of the WMC approach are illustrated in the following sections for a hypothetical fiber.

#### WMC Characterization Applied to a Hypothetical Fiber

To illustrate the WMC characterization a hypothetical three-component fiber will be examined. The three components are a substrate, a CVD sheath, and the fiber surface. Figure 1 schematically shows a cross section of the hypothetical fiber and the three components. Assumed values for component modulus and Weibull parameters are given in Table I. The surface (interface) component at the substrate/CVD sheath interface will be considered to be flaw free and is therefore neglected. A comparison of the  $m$  parameters indicates that the substrate is assumed to be (in STF) a high variance and the CVD sheath a comparatively low variance material (if each were tested alone). No residual stresses or strains are assumed, therefore all  $\epsilon_{u_i} = 0$ . Various fiber diameters will be considered and the lower portion of Table I gives ROM modulus for the hypothetical fiber diameters.

## Weibull Probability Plots

Taking logarithms twice of equation (1) gives:

$$\ln \ln \left[ \frac{1}{1 - G(\epsilon)} \right] = m \ln(\epsilon - \epsilon_u) + \ln(\omega_o^{-m}) \quad (18)$$

This straight line relationship is often used to estimate WSC parameters from experimental fiber tensile data. Some data however are not well approximated by equation (18) (10,22,23). The Weibull probability plot of Figure 2 shows the three components of the hypothetical fiber plotted according to equation (18) for the indicated fiber diameter and test gage length. Using equation (7) and Table I parameters, a plot of  $\ln \ln[1/[1 - Q(\epsilon)]]$  gives the dashed line of Figure 2 for the hypothetical fiber (the three components together). A dual slope ( $m$  parameter) results which is similar to the trends of some plots for experimental tensile data (10,22,23). An envelope is delineated in the upper left hand region of Figure 2 by the substrate and CVD sheath components. That is, at a given probability of failure, the fiber STF is limited by that of the lowest valued component. From the figure it can be deduced that fiber failures at low strain values are dominated by the substrate and by the CVD sheath at high strain values. The line representing the surface component is well outside the envelope and indicates that this component has little effect on fiber STF.

## Effect of Test Gage Length

The effect of test gage length is shown in Figure 3 for a 102  $\mu\text{m}$  (0.004 in.) diameter fiber. Substitution of Table I parameters into equation (3) at various gage lengths generates for each component taken alone, straight lines as shown in Figure 3. For the substrate, CVD sheath, and surface components, respectively, the equations for the lines are:

$$\ln(\bar{\epsilon}_1) = - \frac{1}{m_1} \ln l + \ln \left[ \epsilon_{o1} \left( \frac{\pi d_1^2}{4l} \right)^{-1/m_1} \Gamma \left( \frac{m_1 + 1}{m_1} \right) \right] \quad (19)$$

$$\ln(\bar{\epsilon}_2) = - \frac{1}{m_2} \ln l + \ln \left\{ \epsilon_{o2} \left[ \frac{\pi (d_2^2 - d_1^2)}{4l} \right]^{-1/m_2} \Gamma \left( \frac{m_2 + 1}{m_2} \right) \right\} \quad (20)$$

$$\ln(\bar{\epsilon}_3) = - \frac{1}{m_3} \ln l + \ln \left[ \epsilon_{o3} \left( \frac{\pi d_3^2}{4l} \right)^{-1/m_3} \Gamma \left( \frac{m_3 + 1}{m_3} \right) \right] \quad (21)$$

where  $\alpha$  = defined incremental volume,  $\gamma$  = defined incremental surface area,  $d$  = diameter,  $l$  = gage length, and subscripts refer to the respective components.

Substitution of the same Table I parameters into equation (9) results in the dashed line which represents the composite fiber made up of the three components being considered. From Figure 3 it can be deduced that fiber STF is dominated primarily by CVD sheath flaws for short gage lengths and primarily by substrate flaws at long gage lengths. The surface component line is above and remote from both substrate and CVD sheath lines and indicates that the surface minimally affects fiber STF behavior.

The general shape of the fiber curve in Figure 3 bears resemblance to some experimental results for brittle fibers. The slope changes with gage length and there is a leveling off of mean STF at short gage lengths, termed "roll-off." Roll-off behavior was observed by the investigators of reference 8 (for SiC and sapphire fibers) and by those of reference 10 (for glass fibers). Phoenix and Sexsmith (9) proposed an explanation for apparent roll-off. They theoretically showed that disregarding tensile tests in which fiber failed in the grips could result in an apparent roll-off. Kotchick et al. (8) noted that although part of the roll-off in strength that they observed might be attributed to grip effects, there was an inherent slope change with gage length superimposed on the grip effect. The roll-off in mean STF (and strength) shown in Figure 3 can be attributed to the presence of a dominant component having a mean STF approximately that of the fiber at short gage lengths.

Figure 4 shows, for a 6 cm gage length, probability density curves (calculated by eq. (2)) for each hypothetical fiber component (as if each were tested by itself). Also shown in Figure 4 is the probability density curve for the total fiber as calculated by equation (8). The total fiber curve extends from the lowest STF values of the substrate but does not extend beyond the highest STF values of the CVD sheath. The skewed total fiber curve shape is similar to experimental histograms for some fibers (17,22,24).

Computed values of  $\bar{\epsilon}$ ,  $S_{\epsilon}$ ,  $\bar{\sigma}$ ,  $S_{\sigma}$ , and CV are given in Table II for the hypothetical fiber. Increasing values for standard deviation and CV with gage length are similar to the trends shown for the experimental results of references 8 and 10.

#### Effect of Fiber Diameter

To illustrate effects of composite fiber diameter predicted  $\bar{\epsilon}$  and  $\bar{\sigma}$  values are compiled in Table III for hypothetical fibers made by CVD to the indicated diameters. Component Weibull parameters are again from Table I. Fiber modulus is calculated from the ROM. The computed values in Table III show that for a given gage length  $\bar{\epsilon}$  decreases as fiber diameter is increased and similarly, for a given fiber diameter,  $\bar{\epsilon}$  decreases as gage length increases. Mean UTS values, however, do not necessarily follow the same trends. For the hypothetical fiber being considered,  $\bar{\sigma}$  goes

through a maximum at the various gage lengths. Two opposing effects are operating. As fiber diameter increases, the fiber modulus increases and hence  $\bar{\sigma}$  increases. Ultimately however, as fiber diameter increases, volume effects become important and  $\bar{\sigma}$  decreases.

### Fracture Origin

The expected percent of failures originating in each component is shown in Table IV for a 102  $\mu\text{m}$  (0.004 in.) diameter fiber. Equation (17) was solved by numerical integration methods for 0.5, 2.54, 6, and 30 cm gage lengths for the indicated fiber stress ranges. As was deduced by inspection of Figure 3, the change in fiber fracture origin as a function of test gage length is such that the CVD sheath and substrate dominate fracture behavior at short and long gage lengths, respectively. The surface component contributes only slightly (but not zero) to fiber failures.

### Materials and Experimental Procedures

Longitudinal tensile strength and modulus were determined on samples of the following fiber materials:

**Carbon Fiber Substrate** - This material is a nominally 33  $\mu\text{m}$  (0.0013 in.) diameter carbon fiber and was utilized as a substrate in the CVD manufacture of silicon carbide and carbon-boron alloy fibers described below. As-received and surface etched samples were tensile tested.

**Silicon Carbide (SiC) Fiber** - Two variants of commercially available SiC fiber were tested. One was produced with a 12.7  $\mu\text{m}$  (0.0005 in.) diameter tungsten substrate (SiC/W) and the other with a 33  $\mu\text{m}$  (0.0013 in.) diameter carbon substrate (SiC/C). The carbon-core type is a more recent development and is superior in strength to the tungsten-core fiber. Presumably there is less chemical interaction between the carbon substrate and SiC sheath during high temperature CVD manufacture. The carbon-core fiber has a thin pyrolytic carbon layer, approximately 2.54  $\mu\text{m}$  (0.0001 in.), between the core and the CVD SiC layers. A carbon rich surface layer was applied in the last stages of the CVD manufacturing process to improve resistance to surface damage by handling.

**Carbon-Boron Alloy (CBA) Fiber** - The CBA fiber is an experimental fiber developed and evaluated under NASA contracts (11-15). It is a multicomponent fiber consisting of a carbon substrate (described above) and a single or multiple layered CVD sheath. The CVD sheath was formed by the simultaneous CVD of carbon and boron from appropriate reactant gases onto the electrically heated carbon fiber substrate. Boron content in the sheath ranged from 30 to 65 weight percent. On some runs a thin layer, approximately 2.54  $\mu\text{m}$  (0.0001 in.), of CVD pyrolytic carbon was applied to the substrate as a first step in the manufacturing process. Fiber properties such as elastic modulus, density, and chemical composition were a complex function of fiber draw rate, reactant gas com-

position, reactant gas flow rate, and deposition temperature. Fiber diameter was a function of the same parameters as well as the number and length of CVD reaction chambers in the production line. The integrity and homogeneity of the carbon substrate, as well as all of the above parameters, appeared to be important factors in fiber strength. The fiber samples tested here were produced during performance of the work reported in references 11 to 13. Several examples of the CBA fiber were tested and it was convenient to assign the fiber samples to groups. Table V gives group breakdown together with selected fiber properties. Spools, runs, or lots within a given group were manufactured under similar conditions.

Room temperature tensile tests were performed on a vertical travel testing machine at strain rates of  $0.01 \text{ min}^{-1}$  or  $0.0125 \text{ min}^{-1}$ . Fibers were tested at gage lengths of 0.635 cm (0.25 in.), 2.54 cm (1 in.), 10.16 cm (4 in.), and 25.4 cm (10 in.). At each gage length and for each kind of fiber at least 10 tensile tests were performed. For Groups A and B of the CBA fiber, tensile data (unpublished) obtained by the fiber manufacturer at 0.635 cm (0.25 in.) gage length have been incorporated with results obtained at this laboratory. Carbon substrate and CBA fiber samples were tested using 2.54 cm (1 in.) long grips of epoxy cement contained within rigid grip fixtures. Pneumatic grips faced with aluminum foil were utilized for the SiC samples. In all cases fiber specimens were carefully aligned along the machine tensile axis. No grip failures occurred which could not be attributed to experimental technique.

Static values for longitudinal elastic modulus were determined from tensile test stress-strain data for 25.4 cm (10 in.) gage length specimens. Results were consistently about 5 percent lower than flexural modulus values obtained by vibrating reed dynamic techniques (unpublished research, D. L. McDanel, NASA-Lewis Research Center). Elastic modulus values given in Tables V and VI are dynamic values or 105 percent of static values.

### Results and Discussion

Table VI shows tensile test results for the carbon substrate fiber, the CBA fiber groups, and the two kinds of SiC fiber. Included in Table VI are tensile data from reference 8 for carbon-core, 102  $\mu\text{m}$  (0.004 in.) diameter SiC fiber. Also tabulated are mean STF values (assumed equal to mean UTS divided by fiber modulus). Only elastic strain was considered since no evidence of plastic deformation was observed. Creep deformation was not considered since tensile tests were of short duration and performed at room temperature. ROM calculations approximated experimental SiC fiber moduli when the following component modulus values were assigned: 41.4 GPa (6,000,000 psi) for the carbon substrate and pyrolytic carbon interlayer, 400 GPa (58,000,000 psi) for the tungsten substrate, and 434 GPa (63,000,000 psi) for CVD SiC. Elastic modulus was not reported for the 102  $\mu\text{m}$  (0.004 in.) carbon-core SiC fiber of reference 8 and the value given in Table VI was calculated using the ROM and the above component values.

It is assumed in the following discussion that flaw characterization parameters as well as component modulus, dimensions, and residual strain do not change significantly along the length of a fiber. It is recognized that this assumption is violated somewhat, as these properties can vary from spool to spool as well as along the length of a given spool. However, this should not significantly affect the WMC calculations and trend predictions.

In the WMC characterization the usual Weibull location parameter  $\epsilon_u$  has been defined as residual compressive strain in the  $i^{\text{th}}$  component. The effects of residual strains are not illustrated here, but their consideration would be necessary for fibers having significant residual stresses (e.g., boron-on-tungsten).

In the application of the WMC characterization to the experimental results, little attention was given to minimizing differences in observed and predicted fiber STF or UTS by optimization of flaw parameters. Rather, it is intended to demonstrate that straightforward considerations of known fiber construction and tensile fracture behavior can result in a reasonable description of fiber STF and UTS. In applying the WMC characterization to the STF and UTS behavior of brittle reinforcing fibers, it is suggested that the following procedure will usually provide sufficient information to characterize fiber fracture behavior:

1. Postulate possible components based on known fiber construction and observations of fiber failure origins (e.g., substrate, CVD sheath, etc.).
2. Estimate longitudinal elastic moduli and residual strains for each component.
3. Estimate Weibull parameters for each component from the following considerations:
  - a. Well defined STF or UTS histograms for the fiber, for individual components (e.g., substrate), and for etched fiber (i.e., successive removal of outer layers as performed by Smith (18)).
  - b. Mean STF and UTS behavior as a function of test gage length and fiber diameter. Consideration must be given to clamp effects and to changes in standard deviation and coefficient of variation as a function of test gage length.
  - c. Observations of fiber fracture origins as a function of test gage length and the stress or strain range in which fractures occur.

#### Carbon Fiber Substrate

The data for the carbon fiber substrate can be described, over the range of gage lengths tested, by the WSC characterization with parameters  $m = 4.3$ ,  $\sigma_0 = 0.034$ , and  $\epsilon_u = 0$ . Data are plotted in Figure 5. Although not shown in Figure 5 the tensile



results are in agreement with those of Hough and Richmond (13) who also tested as-received fiber from the same production lot.

#### SiC/Carbon-Core Fiber

Tensile data for SiC/C fiber are plotted in Figure 6. Mean STF for 142  $\mu\text{m}$  (0.0056 in.) fiber is somewhat lower than the computed mean STF for 102  $\mu\text{m}$  (0.004 in.) fiber. Although mean STF decreases with increasing gage length, the data cannot be described by WSC equation (3) since data do not fall on a straight line. Furthermore, the trend of increasing standard deviation and CV with gage length, shown in Table VI, is contrary to predictions of the WSC characterization.

To describe tensile behavior by the WMC approach, the fiber will be considered to consist of four components: substrate, CVD carbon interlayer, CVD SiC sheath, and a carbon-rich surface layer. Estimated properties and assumed Weibull parameters for the components are given in Table VII. Rationale for their selection follows.

Little information is available regarding residual strains in carbon-core SiC although Crane and Krukoni (19) report that residual stresses/strains in tungsten-core SiC fiber are small. For lack of other evidence, residual strains were assumed to be negligible and therefore all  $\epsilon_{ui} = 0$ . The carbon substrate is assumed identical to that tested in this investigation. McHenry and Tressler (20) observed that tensile fractures always seemed to initiate at the carbon/SiC interface or in the carbon core for 102  $\mu\text{m}$  (0.004 in.) SiC/carbon-core fiber. Although it was not indicated, it will be assumed here that these observations pertained to tensile tests at short gage lengths. It is further assumed that fracture origins are in the 2.54  $\mu\text{m}$  (0.0001 in.) thick CVD carbon interlayer. Assumed Weibull parameters given in Table VII correspond to the carbon interlayer line shown in Figure 6. Parameters for the SiC sheath component were selected such that this component would become the dominant contributor to fiber failure at long gage lengths. Finally, Weibull parameters for the carbon-rich surface component were chosen so that this component would contribute very little to fiber fractures. This is consistent with the intent of improving the surface flaw character by application of the carbon-rich surface layer.

The computed mean STF is plotted in Figure 6 for each component for 102  $\mu\text{m}$  and 142  $\mu\text{m}$  diameter fibers. Note that the substrate and CVD carbon interlayer components are common to each diameter fiber. The expected mean STF, equation (9), for 102  $\mu\text{m}$  and 142  $\mu\text{m}$  diameter fibers is also shown.

Table VIII compares predicted and observed (where appropriate) tensile properties for three fiber diameters. Observed and predicted mean UTS are in reasonable agreement. Observed standard deviation and CV are somewhat lower than the predicted values but the trend of increasing magnitudes with test gage length is duplicated.

The WMC characterization is a phenomenological description of fiber tensile behavior and provides no information as to means of improving the flaw character of the various fiber components. It would in principle however, identify the portion of a fiber to which, if improvements were made, improved total fiber performance would result. For example, if the WMC description for the SiC/C fiber is correct, Figure 6 and Table VII, and the fiber were intended for use in a composite system where the critical length is short, then there would be little to be gained by improvement of the flaw character in the CVD SiC sheath or fiber surface components. On the other hand, if the fiber were to be utilized for long gage length applications, sheath and surface improvement would definitely be in order. Figure 6 also indicates that for short gage lengths (less than 1 cm), there is little difference in fiber STF behavior as a function of fiber diameter. At longer lengths (greater than 2 cm) however, fiber diameter would be an important factor in affecting the mean STF.

#### SiC/Tungsten-Core Fiber

This fiber will also be considered to consist of four components as indicated in Table VII. The tungsten core will be considered flaw-free. It is assumed that the core/SiC sheath interface dominates fiber failure and the Weibull parameters chosen correspond to the interface component depicted in Figure 7. That this interface dominates fiber fracture is consistent with observations of references 19 and 25. Weibull parameters for the CVD SiC sheath and carbon-rich surface are identical to those used for the SiC/C fiber discussed previously. No residual strains were assigned to the fiber components since reference 19 reported that residual stresses in SiC fiber are small in magnitude. Observed and calculated mean UTS values are given in Table VIII. Figure 7 shows observed and calculated mean STF.

SiC/W-core tensile properties were reported (19) for 102  $\mu\text{m}$  (0.004 in.) diameter fiber. The fiber was produced prior to incorporation of the carbon-rich surface layer treatment into the manufacturing process. The tensile data were for a 10.4 cm (4 in.) gage length and it was observed that fiber failures initiated predominantly at three sites: the fiber surface, the tungsten-core/SiC sheath interface, and occasionally within the tungsten core. The SiC sheath, was not observed to initiate failures. Of the fractured fiber specimens that could be examined, the following estimates were made regarding fracture origin. Below 2240 MPa (325 ksi) about 75% were surface initiated and 25% were at the tungsten-core/SiC-sheath interface. At strength levels above 3100 MPa (450 ksi), the primary fracture sites, about 95%, were at flaws at the W-core/sheath interface or within the tungsten core. Mixed fracture origin was observed between 2240 and 3100 MPa (350 and 450 ksi). Residual stresses were small in magnitude and are considered negligible here.

To characterize this "old" generation SiC/W fiber, the same components are assumed as above excepting the surface component. The Weibull parameters of the "old" surface component were selected to impose a slightly lower mean STF and greater scatter

(consistent with improvement of fiber properties by application of the carbon-rich layer). Table VII shows assumed component properties and Weibull parameters for this "old" generation fiber. Table IX shows WMC predictions for fiber fracture origin as a function of fiber stress range. Results are reasonably consistent with the observations of reference 19.

### CBA Fiber

Because of the wide variety of conditions and fabrication parameters under which the CBA fiber samples were produced, and because only limited amounts of tensile data for a given run were obtained, no attempt is made to definitively describe tensile behavior in terms of a WMC characterization. Rather, experimental tensile results are discussed in a general sense, as they could be interpreted by the WMC approach.

Tensile data for the CBA fiber groups are shown in the  $\ln \bar{\epsilon}$  versus  $\ln l$  plot in Figure 8. For a given group the WSC characterization appears to satisfactorily describe the data although there is no apparent relationship between groups. Contrary to usual size-strength predictions, the large diameter groups fail at higher strain (on the average) than the small diameter fiber.

For Groups A and B the following is known. Both groups have the following components in common: (1) a carbon substrate of equal volume, (2) a substrate/CVD sheath interface of equal area, (3) equal volumes of CVD sheath, and (4) equal fiber surface areas. The following component properties differ: (1) the CVD sheath differs in modulus and chemical composition, Tables V and VI, and perhaps in flaw distribution as well, and (2) Group A fiber was produced with a 4-chamber CVD apparatus and Group B with a 3-chamber apparatus. References 17 and 19 have indicated that foreign object inclusions can be picked up at CVD chamber entry and exit electrodes. Thus, there is a potential for a strain limiting component at each CVD layer interface, with Group A fiber having the greater interface surface area. The statistical characterization of this type of interface component is difficult since flaws would presumably be a function of the cleanliness of the mercury electrodes at the particular time(s) of fiber manufacture. However, the relative position (lower  $\bar{\epsilon}$ , fig. 8) of the Group A fiber is consistent with the greater CVD layer interface area.

Fiber from Groups C and D are essentially identical except for a small difference in modulus and a proprietary surface treatment given Group D. The primary difference is the surface treatment given Group D fiber. For purposes here it can be considered that this surface treatment heals or negates flaws at the fiber surface, or in effect, Group C fiber has a surface component and Group D has none. The lower mean STF for Group C relative to Group D fibers shown in Figure 8 is consistent with such conjecture.

### Concluding Remarks

A Weibull multicomponent (WMC) characterization for brittle fiber tensile fracture behavior was presented. The WMC characterization provides a framework for the study of brittle fiber tensile behavior as a function of component volume and/or surface area, component modulus, component residual strain, and the STF behavior of each component. The WMC approach has appeal in that component properties and flaw parameters are associated with observable fiber components such as substrate, CVD sheath, surface and interfaces.

The characterization permits an improved brittle fiber strength description. It is believed it will aid in the identification of those portions of a fiber to which, if improvements were made, significant improvements in total fiber performance would (or would not) result. This feature was illustrated as it might apply to a silicon carbide fiber having a carbon substrate.

The WMC approach was illustrated by means of a hypothetical fiber where it was shown that many experimental observations pertaining to brittle fiber tensile testing can be simulated. The WMC characterization can simulate and/or account for highly skewed strength histograms, strength roll-off with decreasing gage length, and changes in dominant fracture locations as a function of gage length and/or stress range.

Tensile data for silicon carbide fiber were interpreted in terms of the WMC characterization. Changes in mean STF (and strength) and standard deviation as a function of gage length and fiber diameter were closely approximated.

### Nomenclature

|   |                                                                          |
|---|--------------------------------------------------------------------------|
| d | diameter                                                                 |
| E | longitudinal elastic modulus                                             |
| G | cumulative distribution function for failure of a single component fiber |
| g | probability density function for a single component fiber                |
| l | gage length                                                              |
| m | Weibull shape parameter                                                  |
| Q | cumulative distribution function for failure of a multicomponent fiber   |
| q | probability density function for a multicomponent fiber                  |
| S | standard deviation                                                       |
| V | volume percent                                                           |

|                  |                                                                                                                                                                       |
|------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $Z_j$            | expected fraction of fiber failures from a large sample whose fracture origin was in the $j^{\text{th}}$ component and occurred between $\epsilon_1$ and $\epsilon_2$ |
| $\alpha$         | defined incremental volume, $1.6387 \times 10^{-5} \text{ cm}^3$ ( $10^{-6} \text{ in}^3$ )                                                                           |
| $\gamma$         | defined incremental surface area, $6.4516 \times 10^{-3} \text{ cm}^2$ ( $10^{-3} \text{ in}^2$ )                                                                     |
| $\epsilon$       | strain to failure                                                                                                                                                     |
| $\bar{\epsilon}$ | mean strain to failure                                                                                                                                                |
| $\epsilon_o$     | Weibull scale parameter                                                                                                                                               |
| $\epsilon_u$     | Weibull location parameter                                                                                                                                            |
| $\sigma$         | fiber ultimate tensile stress                                                                                                                                         |
| $\bar{\sigma}$   | mean ultimate tensile stress                                                                                                                                          |
| $\omega$         | number of incremental volumes or surfaces in a test gage length                                                                                                       |

### Subscripts

|            |                       |
|------------|-----------------------|
| f          | fiber                 |
| i          | component designation |
| j          | component designation |
| $\epsilon$ | STF                   |
| $\sigma$   | fiber UTS             |

### Acronyms

|       |                                             |
|-------|---------------------------------------------|
| CBA   | carbon-boron alloy fiber                    |
| CV    | coefficient of variation                    |
| CVD   | chemical vapor deposition                   |
| ROM   | rule-of-mixtures                            |
| SiC/C | silicon carbide fiber on carbon substrate   |
| SiC/W | silicon carbide fiber on tungsten substrate |
| STF   | strain-to-failure                           |
| UTS   | ultimate tensile strength                   |

WMC Weibull multicomponent

WSC Weibull single-component

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TABLE I. - ASSIGNED PROPERTIES AND WEIBULL FLAW PARAMETERS FOR THE HYPOTHETICAL FIBER

| Number | Component     | Diameter,<br>$\mu\text{m}$ (mil) | Elastic<br>modulus,<br>GPa (Mpsi) | Residual<br>compressive<br>strain,<br>$\epsilon_u$ | Weibull parameters<br>m | $\epsilon_o$ |
|--------|---------------|----------------------------------|-----------------------------------|----------------------------------------------------|-------------------------|--------------|
| 1      | Substrate     | 33.0 (1.3)                       | 41.37 (6.0)                       | 0                                                  | 3.0                     | 0.020        |
| 2      | CVD sheath    | $a_{d_2}$                        | 206.8 (30.0)                      | 0                                                  | 25.0                    | .014         |
| 3      | Fiber surface | $a_{d_3}$                        | -----                             | 0                                                  | 7.0                     | .034         |
|        | Fiber         | 50.8 (2.0)                       | 137.0 (19.86)                     |                                                    |                         |              |
|        | Fiber         | 86.4 (3.4)                       | 183.0 (26.49)                     |                                                    |                         |              |
|        | Fiber         | 102.0 (4.0)                      | 189.0 (27.47)                     |                                                    |                         |              |
|        | Fiber         | 142.0 (5.6)                      | 198.0 (28.71)                     |                                                    |                         |              |
|        | Fiber         | 203.0 (8.0)                      | 202.0 (29.37)                     |                                                    |                         |              |
|        | Fiber         | 279.0 (11.0)                     | 205.0 (29.66)                     |                                                    |                         |              |
|        | Fiber         | 406.0 (16.0)                     | 206.0 (29.84)                     |                                                    |                         |              |

<sup>a</sup>Sheath and surface diameter will be that of whatever fiber is being considered.

TABLE II. - PREDICTED VALUES FOR THE PROPERTIES OF THE HYPOTHETICAL FIBER AS A FUNCTION OF TEST GAGE LENGTH

[Fiber diameter is 102  $\mu\text{m}$  (0.004 in.).]

| Fiber property               | Gage length, cm |            |            |            |
|------------------------------|-----------------|------------|------------|------------|
|                              | 0.5             | 2.54       | 6          | 30         |
| $\bar{\epsilon}$             | 0.01301         | 0.01150    | 0.01028    | 0.00705    |
| $S_{\bar{\epsilon}}$         | 0.00130         | 0.00203    | 0.00245    | 0.00244    |
| $\bar{\sigma}$ MPa (ksi)     | 2460 (357)      | 2180 (316) | 1950 (282) | 1330 (194) |
| $S_{\bar{\sigma}}$ MPa (ksi) | 245 (35.6)      | 385 (55.8) | 463 (67.2) | 461 (66.8) |
| Coefficient of variation     | 0.100           | 0.177      | 0.238      | 0.345      |

TABLE III. - PREDICTED MEAN STP AND UTS AS A FUNCTION OF DIAMETER AND GAGE LENGTH FOR THE HYPOTHETICAL FIBER

| Fiber<br>diameter,<br>$\mu\text{m}$ (mil) | Elastic<br>modulus,<br>GPa (Mpsi) | Gage length, cm           |                             |                           |                             |                           |                             |                           |                             |
|-------------------------------------------|-----------------------------------|---------------------------|-----------------------------|---------------------------|-----------------------------|---------------------------|-----------------------------|---------------------------|-----------------------------|
|                                           |                                   | 0.5                       |                             | 2.54                      |                             | 6                         |                             | 30                        |                             |
|                                           |                                   | $\bar{\epsilon}$<br>cm/cm | $\bar{\sigma}$<br>MPa (ksi) | $\bar{\epsilon}$<br>cm/cm | $\bar{\sigma}$<br>MPa (ksi) | $\bar{\epsilon}$<br>cm/cm | $\bar{\sigma}$<br>MPa (ksi) | $\bar{\epsilon}$<br>cm/cm | $\bar{\sigma}$<br>MPa (ksi) |
| 33.0 (1.3)                                | 41.4 (6.0)                        | 0.02794                   | 1160 (168)                  | 0.01625                   | 672 (98)                    | 0.01220                   | 505 (73)                    | 0.00714                   | 295 (43)                    |
| 50.8 (2.0)                                | 137 (19.86)                       | 0.01393                   | 1910 (277)                  | 0.01216                   | 1670 (242)                  | 0.01070                   | 1470 (213)                  | 0.00709                   | 972 (141)                   |
| 86.4 (3.4)                                | 183 (26.49)                       | 0.01320                   | 2410 (350)                  | 0.01164                   | 2130 (308)                  | 0.01037                   | 1900 (275)                  | 0.00706                   | 1290 (187)                  |
| 102 (4.0)                                 | 189 (27.47)                       | 0.01301                   | 2460 (357)                  | 0.01149                   | 2180 (316)                  | 0.01028                   | 1950 (282)                  | 0.00705                   | 1340 (194)                  |
| 142 (5.6)                                 | 198 (28.71)                       | 0.01266                   | 2510 (364)                  | 0.01124                   | 2230 (323)                  | 0.01010                   | 2000 (290)                  | 0.00702                   | 1390 (202)                  |
| 203 (8.0)                                 | 202 (29.37)                       | 0.01231                   | 2490 (361)                  | 0.01097                   | 2220 (322)                  | 0.00991                   | 2010 (291)                  | 0.00698                   | 1420 (205)                  |
| 279 (11.0)                                | 205 (29.66)                       | 0.01200                   | 2460 (356)                  | 0.01074                   | 2200 (319)                  | 0.00974                   | 2000 (289)                  | 0.00695                   | 1430 (206)                  |
| 406 (16.0)                                | 206 (29.84)                       | 0.01166                   | 2400 (348)                  | 0.01047                   | 2160 (312)                  | 0.00953                   | 1970 (284)                  | 0.00690                   | 1430 (206)                  |



TABLE IV. - EXPECTED PERCENTAGE OF TOTAL FIBER TENSILE FAILURE WHOSE ORIGIN WAS IN THE NOTED COMPONENT AND FIBER FOR THE INDICATED STRESS RANGES

[Computed percentages are for a hypothetical 102  $\mu$ m (0.004 in.) diameter fiber at the test gage lengths shown.]

| Gage length, cm | Fracture origin | Range of fiber stress, MPa (ksi) |                            |                            |                                 |
|-----------------|-----------------|----------------------------------|----------------------------|----------------------------|---------------------------------|
|                 |                 | 0 - 1030<br>(0 - 150)            | 1030 - 2410<br>(150 - 350) | Greater than<br>2410 (350) | Entire range<br>(0 - $\infty$ ) |
| 0.5             | Substrate       | 0.5                              | 5.9                        | 1.0                        | 7.4                             |
|                 | CVD sheath      | 0                                | 17.9                       | 74.4                       | 92.3                            |
|                 | Fiber surface   | 0                                | .2                         | .1                         | .3                              |
|                 | Fiber           | 0.5                              | 24.0                       | 75.5                       | 100.0                           |
| 2.54            | Substrate       | 2.7                              | 24.1                       | 0.6                        | 27.3                            |
|                 | CVD sheath      | 0                                | 48.4                       | 23.3                       | 71.8                            |
|                 | Fiber surface   | 0                                | .9                         | .1                         | .9                              |
|                 | Fiber           | 2.7                              | 73.3                       | 24.0                       | 100.0                           |
| 6.0             | Substrate       | 6.2                              | 42.8                       | 0.1                        | 49.1                            |
|                 | CVD sheath      | 0                                | 46.3                       | 3.3                        | 49.6                            |
|                 | Fiber surface   | 0                                | 1.3                        | .0                         | 1.3                             |
|                 | Fiber           | 6.2                              | 90.4                       | 3.4                        | 100.0                           |
| 30.0            | Substrate       | 27.3                             | 65.5                       | 0                          | 92.8                            |
|                 | CVD sheath      | 0                                | 6.1                        | 0                          | 6.1                             |
|                 | Fiber surface   | 0                                | 1.1                        | 0                          | 1.1                             |
|                 | Fiber           | 27.4                             | 72.6                       | 0                          | 100.0                           |

TABLE V. - CHARACTERISTICS AND PROPERTIES OF THE CARBON-BORON  
ALLOY (CBA) FIBER GROUPS

|                                                    | Group             |                                   |            |                    |
|----------------------------------------------------|-------------------|-----------------------------------|------------|--------------------|
|                                                    | A                 | B                                 | C          | D                  |
| Manufacturers<br>spool, run, lot<br>identification | 1040, P69,<br>P70 | P31, P57,<br>P62, Run I,<br>Run G | P498       | <sup>a</sup> P499B |
| Diameter,<br>$\mu\text{m}$ (mils)                  | 86.4 (3.40)       | 86.4 (3.40)                       | 140 (5.52) | 140 (5.52)         |
| Carbon<br>interlayer                               | No                | No                                | Yes        | Yes                |
| Number of CVD<br>reaction<br>chambers              | 4                 | 3                                 | 3          | 3                  |
| Weight percent<br>boron <sup>b</sup>               | 43 - 46           | 35 - 39                           | 46 - 48    | 48 - 51            |
| Fiber density, <sup>b</sup><br>g/cm                | 1.96              | 1.84                              | 1.97       | 2.00               |
| Fiber modulus, <sup>b</sup><br>GPa (Mpsi)          | 177 (25.7)        | 129 (18.7)                        | 199 (28.9) | 210 (30.4)         |

<sup>a</sup>Proprietary surface treatment applied to fiber for strength improvement.

<sup>b</sup>Unpublished research, D. L. McDanels, NASA Lewis Research Center.

TABLE VI. - MECHANICAL PROPERTIES OF VARIOUS FIBERS

|                                            | Nominal<br>diameter,<br>μm (mil) | Elastic<br>modulus,<br>GPa (Mpsi) | Gage length,<br>cm (in.) | Mean UTS,<br>MPa (kpsi) | Standard<br>deviation<br>of UTS,<br>MPa (kpsi) | Mean<br>STF,<br>cm/cm | Coefficient<br>of<br>variation |
|--------------------------------------------|----------------------------------|-----------------------------------|--------------------------|-------------------------|------------------------------------------------|-----------------------|--------------------------------|
| Carbon fiber<br>substrate<br>(as-received) | 33 (1.30)                        | 41.4 (6.0)                        | 2.54 (1.0)               | 1280 (186)              | 283 (41)                                       | 0.03100               | 0.220                          |
|                                            |                                  |                                   | 10.16 (4.0)              | 848 (123)               | 124 (18)                                       | .02050                | .146                           |
|                                            |                                  |                                   | 25.4 (10.0)              | 786 (114)               | 221 (32)                                       | .01900                | .281                           |
| Carbon fiber<br>substrate<br>(etched)      | 27 (1.05)                        | 37.2 (5.4)                        | 2.54 (1.0)               | 1170 (170)              | 131 (19)                                       | 0.03148               | 0.112                          |
|                                            |                                  |                                   | 10.16 (4.0)              | 903 (131)               | 207 (30)                                       | .02426                | .229                           |
|                                            |                                  |                                   | 25.4 (10.0)              | 634 (92)                | 276 (40)                                       | .01704                | .435                           |
| SiC/C-Core                                 | 142 (5.60)                       | 406 (58.9)                        | 2.54 (1.0)               | 4220 (612)              | 862 (125)                                      | 0.01039               | 0.204                          |
|                                            |                                  |                                   | 10.16 (4.0)              | 3360 (488)              | 924 (134)                                      | .00828                | .275                           |
|                                            |                                  |                                   | 25.4 (10.0)              | 2690 (390)              | 862 (125)                                      | .00662                | .321                           |
| SiC/C-Core <sup>a</sup>                    | 102 (4.00)                       | 379 (55.0)                        | 0.635 (0.25)             | 5060 (734)              | 283 (41)                                       | 0.01335               | 0.056                          |
|                                            |                                  |                                   | 1.78 (.5)                | 4900 (711)              | 338 (49)                                       | .01293                | .069                           |
|                                            |                                  |                                   | 2.54 (1.0)               | 4790 (696)              | 296 (43)                                       | .01266                | .062                           |
|                                            |                                  |                                   | 5.08 (2.0)               | 4340 (630)              | 607 (88)                                       | .01146                | .140                           |
|                                            |                                  |                                   | 7.62 (3.0)               | 3780 (548)              | 593 (86)                                       | .00997                | .157                           |
|                                            |                                  |                                   | 25.4 (10.0)              | 2850 (413)              | 669 (97)                                       | .00751                | .235                           |
|                                            |                                  |                                   | 50.8 (20.0)              | 2590 (375)              | 662 (96)                                       | .00682                | .256                           |
|                                            |                                  |                                   | 7.62 (3.0)               | 3920 (569)              | 648 (94)                                       | .01035                | .165                           |
|                                            |                                  |                                   | 25.4 (10.0)              | 2960 (430)              | 648 (94)                                       | .00782                | .219                           |
| SiC/W-Core                                 | 142 (5.60)                       | 434 (63.0)                        | 2.54 (1.0)               | 3470 (504)              | 172 (25)                                       | 0.00801               | 0.050                          |
|                                            |                                  |                                   | 25.4 (10.0)              | 2900 (420)              | 441 (64)                                       | .00667                | .152                           |
| CBA Group A                                | 86.4 (3.40)                      | 177 (25.7)                        | 0.635 (0.25)             | 2400 (348)              | 407 (59)                                       | 0.01354               | 0.170                          |
|                                            |                                  |                                   | 2.54 (1.0)               | 1590 (231)              | 558 (81)                                       | .00899                | .351                           |
|                                            |                                  |                                   | 10.16 (4.0)              | 1100 (160)              | 83 (12)                                        | .00623                | .075                           |
|                                            |                                  |                                   | 25.4 (10.0)              | 841 (122)               | 138 (20)                                       | .00475                | .164                           |
| CBA Group B                                | 86.4 (3.40)                      | 129 (18.7)                        | 0.635 (0.25)             | 2320 (337)              | 345 (50)                                       | 0.01802               | 0.148                          |
|                                            |                                  |                                   | 2.54 (1.0)               | 1630 (237)              | 462 (67)                                       | .01267                | .283                           |
|                                            |                                  |                                   | 10.16 (4.0)              | 1230 (179)              | 310 (45)                                       | .00957                | .251                           |
|                                            |                                  |                                   | 25.4 (10.0)              | 1080 (157)              | 179 (26)                                       | .00840                | .166                           |
| CBA Group C                                | 140 (5.52)                       | 199 (28.9)                        | 2.54 (1.0)               | 2780 (403)              | 621 (90)                                       | 0.01394               | 0.223                          |
|                                            |                                  |                                   | 25.4 (10.0)              | 1890 (274)              | 476 (69)                                       | .00948                | .252                           |
| CBA Group D                                | 140 (5.52)                       | 210 (30.4)                        | 2.54 (1.0)               | 3740 (543)              | 724 (105)                                      | 0.01879               | 0.193                          |
|                                            |                                  |                                   | 25.4 (10.0)              | 3280 (475)              | 165 (24)                                       | .01644                | .051                           |

<sup>a</sup>Tensile data from ref. 8.<sup>b</sup>Calculated from rule-of-mixtures.

TABLE VII. - ASSIGNED PROPERTIES AND WEIBULL FLAW PARAMETERS FOR POSTULATED COMPONENTS OF SiC FIBER

| Fiber           | Number | Component             | Diameter,<br>μm (mil)       | Elastic<br>modulus,<br>GPa (Mpsi) | Residual<br>compressive<br>strain,<br>ε <sub>0</sub> | Weibull parameters<br>m      σ <sub>0</sub> |
|-----------------|--------|-----------------------|-----------------------------|-----------------------------------|------------------------------------------------------|---------------------------------------------|
| SiC/C           | 1      | Carbon core           | 33.0 (1.3)                  | 41.2 (6.0)                        | 0                                                    | 4.3    0.034                                |
|                 | 2      | CVD carbon interlayer | 38.1 (1.5)                  | 41.2 (6.0)                        | 0                                                    | 22.0   .013                                 |
|                 | 3      | CVD SiC sheath        | <sup>a</sup> d <sub>s</sub> | 434 (63.0)                        | 0                                                    | 3.6    .036                                 |
|                 | 4      | Carbon-rich surface   | <sup>a</sup> d <sub>s</sub> | -----                             | 0                                                    | 5.0    .030                                 |
| SiC/W (current) | 1      | Tungsten core         | 12.7 (0.5)                  | 400 (58.0)                        | 0                                                    | -----                                       |
|                 | 2      | Core/sheath interface | 12.7 (.5)                   | -----                             | 0                                                    | 45.0   0.00834                              |
|                 | 3      | CVD SiC sheath        | 142 (5.6)                   | 434 (63.0)                        | 0                                                    | 3.6    .036                                 |
|                 | 4      | Carbon-rich surface   | 142 (5.6)                   | -----                             | 0                                                    | 5.0    .030                                 |
| SiC/W (old)     | 1      | Tungsten core         | 12.7 (0.5)                  | 400 (58.0)                        | 0                                                    | -----                                       |
|                 | 2      | Core/sheath interface | 12.7 (.5)                   | -----                             | 0                                                    | 45.0   0.00834                              |
|                 | 3      | CVD SiC sheath        | 102 (4.0)                   | 434 (63.0)                        | 0                                                    | 3.6    .036                                 |
|                 | 4      | "Old" surface         | 102 (4.0)                   | -----                             | 0                                                    | 3.0    .0327                                |

<sup>a</sup>Diameter will be that of whatever fiber is being considered.

TABLE VIII. - COMPARISON OF OBSERVED AND WMC PREDICTED TENSILE STRENGTH FOR SiC FIBERS

| Fiber I.D.                     | Gage length,<br>cm (in.) |        | Mean UTS,<br>ksi <sup>a</sup> |           | Standard deviation<br>of UTS, ksi <sup>a</sup> |           | Coefficient of<br>variation |           |
|--------------------------------|--------------------------|--------|-------------------------------|-----------|------------------------------------------------|-----------|-----------------------------|-----------|
|                                |                          |        | Observed                      | Predicted | Observed                                       | Predicted | Observed                    | Predicted |
| 102 $\mu$ m (4 mil)            | 0.635                    | (0.25) | 734                           | 746       | 41                                             | 85        | 0.056                       | 0.114     |
| diameter                       | 1.78                     | (.5)   | 711                           | 706       | 49                                             | 99        | .069                        | .140      |
| SiC/carbon-                    | 2.54                     | (1.0)  | 696                           | 659       | 43                                             | 114       | .062                        | .173      |
| core fiber <sup>b</sup>        | 5.08                     | (2.0)  | 630                           | 601       | 88                                             | 126       | .140                        | .210      |
|                                | 7.62                     | (3.0)  | 548                           | 562       | 86                                             | 130       | .157                        | .231      |
|                                | 25.4                     | (10.0) | 413                           | 436       | 97                                             | 120       | .235                        | .276      |
|                                | 50.8                     | (20.0) | 375                           | 368       | 96                                             | 105       | .256                        | .285      |
|                                | 7.62                     | (3.0)  | 569                           | 562       | 94                                             | 130       | .165                        | .231      |
|                                | 25.4                     | (10.0) | 430                           | 436       | 94                                             | 120       | .219                        | .276      |
| 142 $\mu$ m (5.6 mil) diameter | 0.635                    | (0.25) | ---                           | 780       | ---                                            | 112       | -----                       | 0.144     |
| SiC/carbon-                    | 2.54                     | (1.0)  | 612                           | 660       | 125                                            | 142       | 0.204                       | .215      |
| core fiber                     | 10.16                    | (4.0)  | 488                           | 501       | 134                                            | 139       | .275                        | .277      |
|                                | 25.4                     | (10.0) | 390                           | 399       | 125                                            | 116       | .321                        | .291      |
| 204 $\mu$ m (8 mil) diameter   | 0.635                    | (0.25) | ---                           | 770       | ---                                            | 142       | -----                       | 0.184     |
| SiC/carbon-                    | 2.54                     | (1.0)  | ---                           | 613       | ---                                            | 159       | -----                       | .259      |
| core fiber <sup>c</sup>        | 10.16                    | (4.0)  | ---                           | 439       | ---                                            | 129       | -----                       | .294      |
|                                | 25.4                     | (10.0) | ---                           | 344       | ---                                            | 103       | -----                       | .299      |
| 142 $\mu$ m (5.6 mil) diameter | 2.54                     | (1.0)  | 504                           | 499       | 25                                             | 51        | 0.050                       | 0.103     |
| SiC/tungsten-                  | 25.4                     | (10.0) | 420                           | 399       | 64                                             | 95        | .152                        | .238      |
| core fiber                     |                          |        |                               |           |                                                |           |                             |           |

<sup>a</sup>1 ksi = 6.8948 MPa.<sup>b</sup>Tensile data from ref. 8.<sup>c</sup>No SiC/C fiber of this diameter was available for testing (predicted strengths only)TABLE IX. - PREDICTED PERCENTAGE OF TOTAL FIBER FRACTURES WHOSE ORIGIN WAS IN THE INDICATED COMPONENT FOR "OLD" SiC/W-CORE FIBER OF 102  $\mu$ m (0.004 in.) DIAMETER

[Predicted values are for a 10.16 cm (4 in.) gage length.]

| Component             | Stress range                  |                                    |                                       |                                        |
|-----------------------|-------------------------------|------------------------------------|---------------------------------------|----------------------------------------|
|                       | 0 - 2240 MPa<br>(0 - 325 ksi) | 2240 - 3100 MPa<br>(325 - 450 ksi) | Greater than<br>3100 MPa<br>(450 ksi) | Entire stress<br>range (0 - $\infty$ ) |
| Tungsten core         | 0                             | 0                                  | 0                                     | 0                                      |
| Core/sheath interface | 0                             | .3                                 | 39.5                                  | 39.8                                   |
| CVD SiC sheath        | 4.0                           | 6.4                                | 2.9                                   | 13.3                                   |
| "Old" surface         | 17.6                          | 20.9                               | 8.3                                   | 46.9                                   |
| Fiber                 | 21.6                          | 27.7                               | 50.7                                  | 100.0                                  |

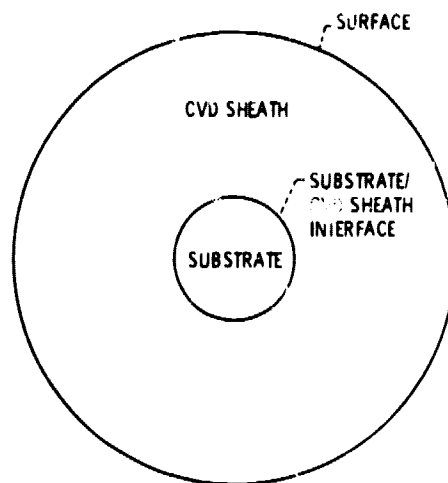


Figure 1. - Schematic cross-section of the hypothetical fiber made by chemical vapor deposition.

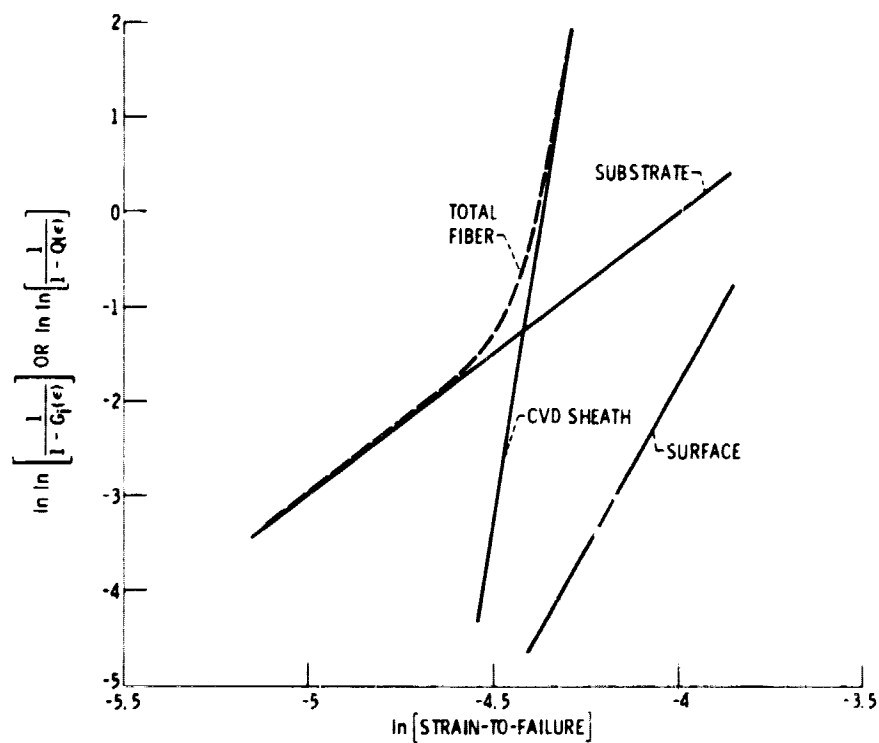


Figure 2. - Weibull probability plots for the hypothetical fiber. The three components are shown separately and taken together as a fiber. Plotted lines are for a 2.54 cm (1 in.) gage length and a 102  $\mu$ m (0.004 in.) fiber diameter.

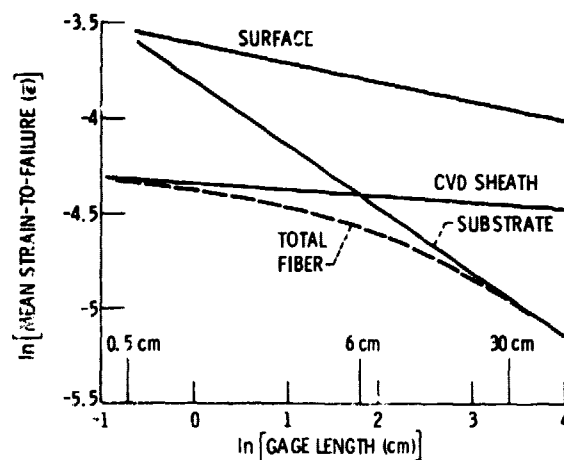


Figure 3. - Mean STF as a function of test gage length for the hypothetical fiber and for its three components taken separately.

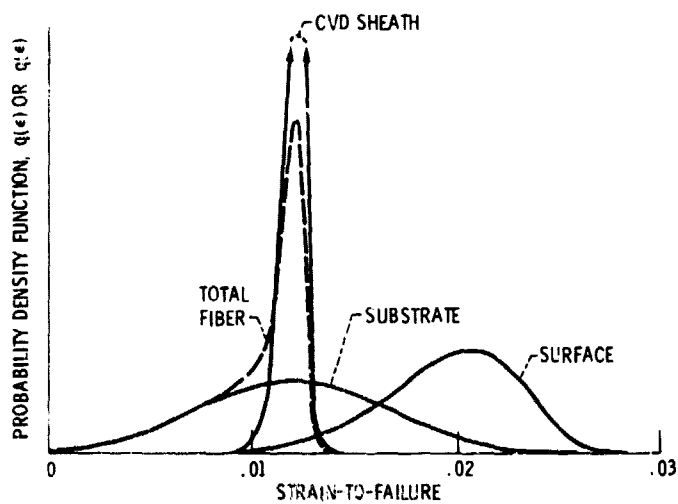


Figure 4. - Probability density curves for a total fiber and for each component separately. Curves are for a hypothetical fiber of diameter 102  $\mu$ m (0.004 in.) and at a gage length of 6 cm.

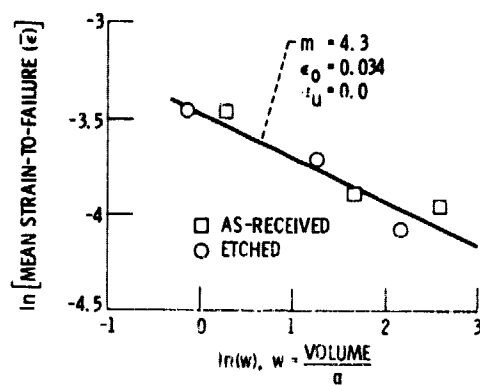


Figure 5. - Mean STF for carbon substrate fiber as a function of test section volume.

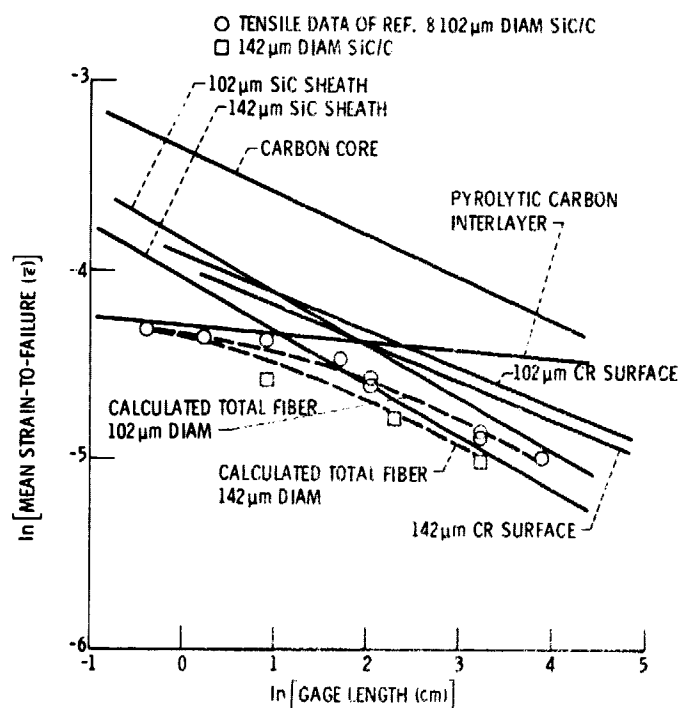


Figure 6. - Calculated mean STF for 102μm and 142μm diameter SiC/C fiber and components as a function of test gage length. Tensile data of reference 8 and of this work are shown.

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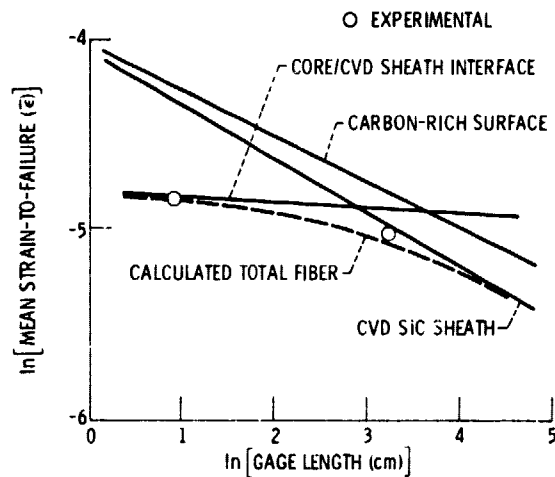


Figure 7. - Calculated mean STF for 142  $\mu\text{m}$  diameter SiC/W fiber as a function of test gage length. Predictions for components and of the fiber are shown together with experimental test results.

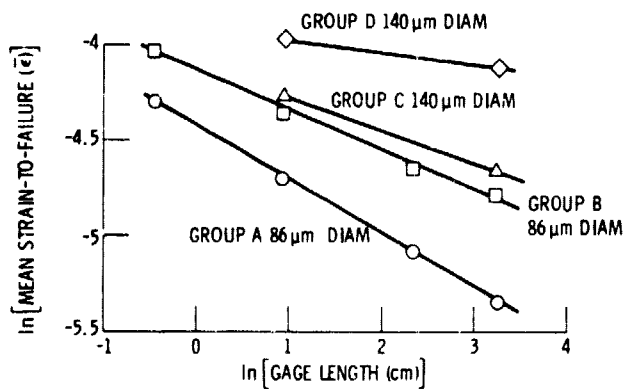


Figure 8. - Observed mean STF for CBA fiber groups as a function of test gage length.